

CS-570 Statistical Signal Processing

Lecture 1: Introduction to CS-570

Spring Semester 2019

Grigorios Tsagkatakis





Today's Objectives

CS-570 Overview

• Introduction to Statistical Signal Processing



Spring Semester 2017-2018



About CS-570

• Lectures

Spring Semester 2017-2018

- ➢ Monday 12:00-14:00, H208
- ➤Wednesday 14:00-16:00, H204
- Office Hours: 1 Hour after each class
- Prerequisites: Digital Signal Processing (CS-370), Applied Mathematics for Engineers (CS-215), Probabilities (CS-217)





About CS-570

Practical Information

- 2 individual homeworks on the material taught (30% of your final grade)
 - Exercises on MATLAB/python
 - 1st assignment will be handed out at the beginning of March
 - 2 weeks time to complete each assignment (hard deadline).
- Standalone project (max for 2 students) (50% of your final grade)
 - Research topic
 - Experimental work / analysis on experimental data
 - Submission of a project report in a technical paper form (motivation, related work, problem formulation, adopted methodology, results, conclusions & outlook)
 - Duration: mid of April End of semester (~mid of June)
- Written Exam (20% of your final grade)

All above are compulsory for getting a grade at the end of the exam





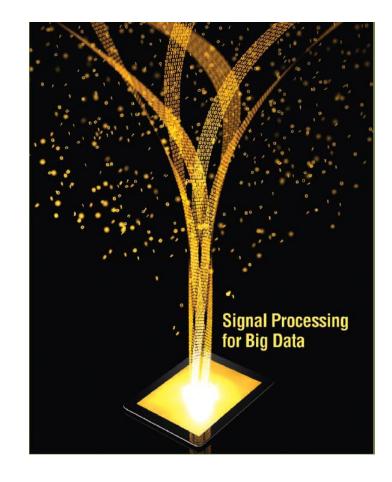
Topics

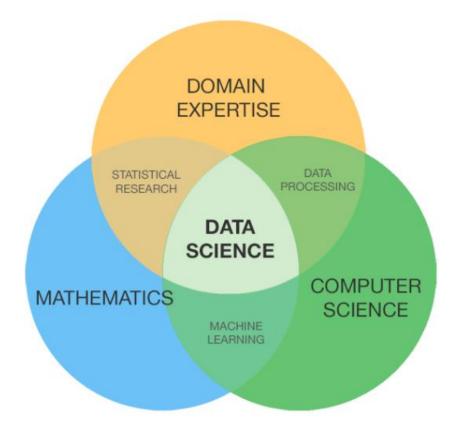
- Week 1: Introduction to Statistical Signal Processing & Review
- Week 2: Introduction to optimization
- Week 3: Signal sensing and reconstruction
- Week 4: Computational imaging
- Week 5: Deterministic signal processing
- Week 6: High and low dimensional signal processing
- Week 7: Statistical signal models
- Week 8: Time-series modeling
- Week 9: Distributed signal processing
- Week 10: Machine learning for signal processing
- Week 11: Applications in remote sensing
- Week 12: Applications in Internet-of-Things

Week 13: Review and presentations



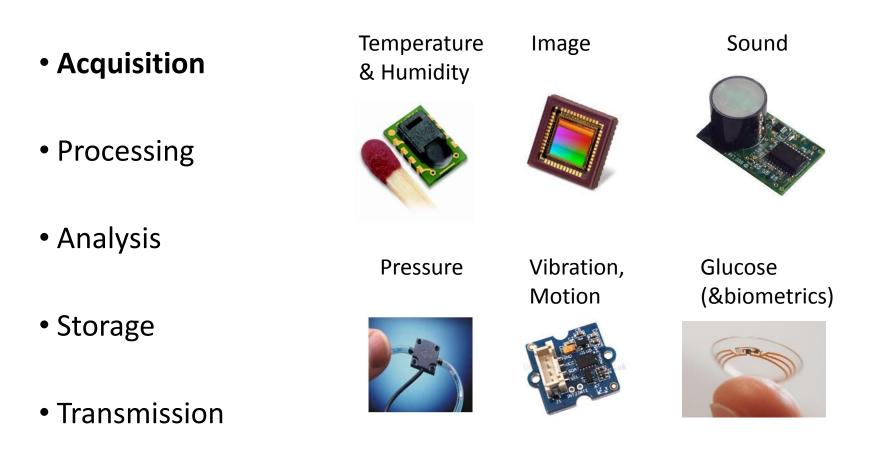












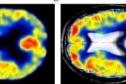




- Acquisition
- Processing
- Analysis
- Storage

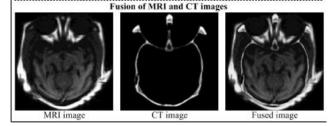
- - Fusion of MRI and PET images







Fused image

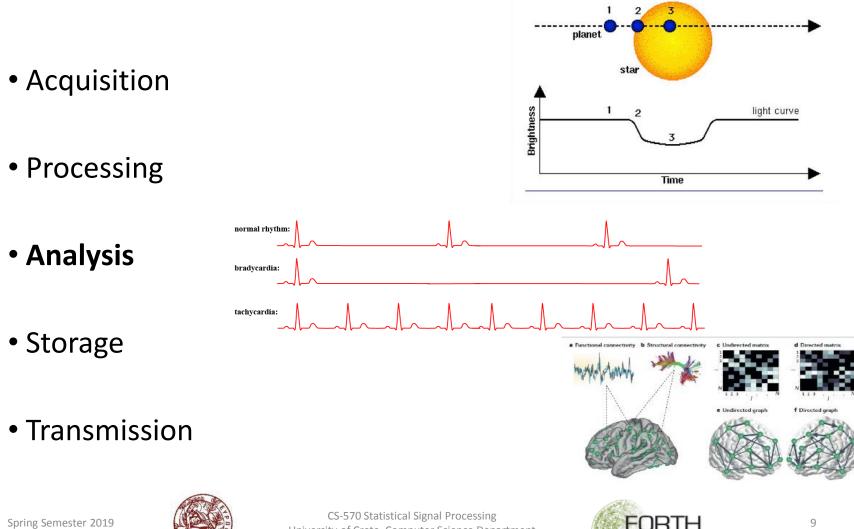




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- Acquisition
- Processing
- Analysis
- Storage

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Transmission





- Acquisition
- Processing
- Analysis
- Storage
- Transmission

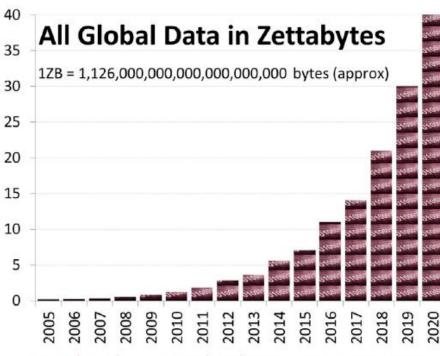








The 5Vs ➢Volume



The growth in data as seen by United Nations Economic Commission for Europe.



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byte sizes byte (B) S D A equivalent to a single character or symbol kilobyte (KB) equivalent to a very short story megabyte (MB) equivalent to a 3.5 inch floppy disk gigabyte (GB) equivalent to 341 average sized digital pictures terabyte (TB) equivalent to a modern day hard drive petabyte (PB) equivalent to 1.5 million **CD-ROM** discs equivalent to 11 million 4K movies zettabyte (ZB) equivalent to 281 trillion MP3 audio files yottabyte (YB) equivalent to 250 trillion





DVDs

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The 5Vs ≻Volume ≻Velocity

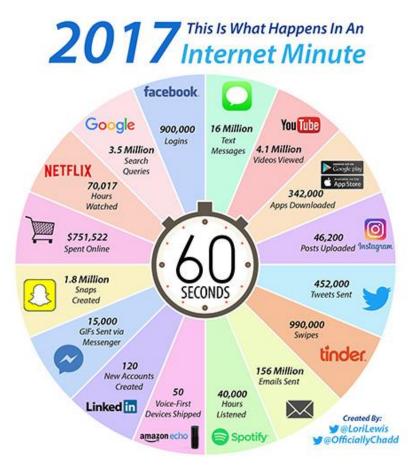
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2016 What happens in an INTERNET MINUTE? facebook. NETFLIX WhatsApp 69,444 701,389 Hours Facebook **150 MILLION** 20.8 MILLION+ **Emails Sent** You Tube Messages UBER 1,389 2.78 MILLION Uber Rides tinder 972,222 ß 527,760 Photos Shared 51,000 App Downloads From Apple 2.4 MILLION Search Queries SECONDS Available on the App Store Google 38,052 Hours of Music \$203,596 120+ amazon Spotify New Linkedin Accounts 1.04 MILLION Vine Loops 38,194 Posts to Instagram 347,222 Vine Linked in EXCELACOM @2016 Excelacom, Inc.





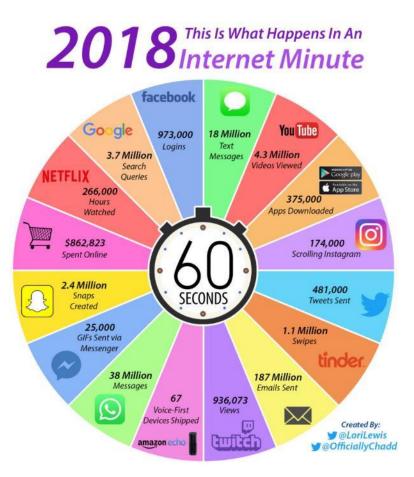
The 5Vs ≻Volume ≻Velocity







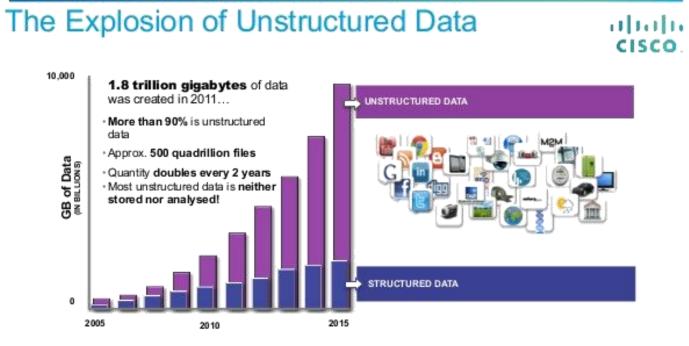
The 5Vs ≻Volume **≻Velocity**







The 5Vs ≻Volume >Velocity >Variety



Source: Cloudera

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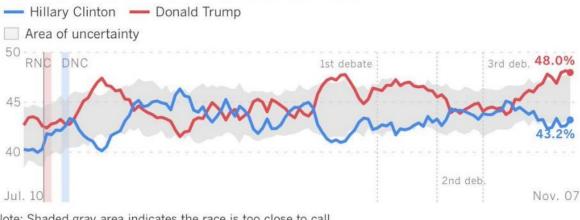


The 5Vs →Volume →Velocity →Variety →Veracity

Who's Winning? Daily track of Clinton and Trump's support

Updated daily.

More from the poll, and why it differs from others.

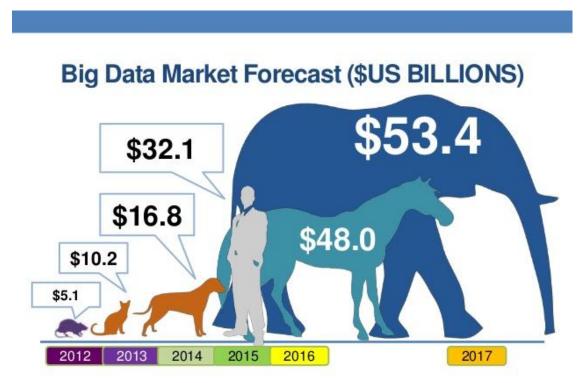


Note: Shaded gray area indicates the race is too close to call. Sources: USC Dornsife/LA Times Presidential Election Daybreak Poll





The 5Vs →Volume →Velocity →Variety →Veracity →Value



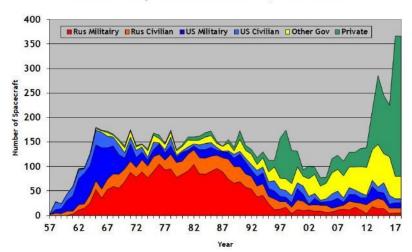




Earth Observation

The 6 Copernicus Sentinels are producing **over 12 TB** of high-quality full, free and open Earth Observation data **every day**, the equivalent of 6,000+ DVDs

Number of Spacecraft Launched, 1957-2017



Volume of data distributed: 42 PB

42 PB



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Astrophysics

Sky Survey Project	Volume	Velocity
Sloan Digital Sky Survey (SDSS)	50 TB	200 GB per day
Large Synoptic Survey Telescope (LSST)	~ 200 PB	10 TB per day
Square Kilometer Array (SKA)	~ 4.6 EB	150 TB per day

THE BIGGER THE BETTER





HOOKER (100") Mt Wilson, California (1917)



(Large Altazimuth Telescope) Zelenchuksky, Russia (1975)



(1979-1998) (1979-1998) MULTI MIRROR TELESCOPE Mt Hopkins, Arizona



KECK TELESCOPE

GEMINI SOUTH

Cerro Pachón

Chile (2000)

Las Campanas Chile

(2000/2002)

LARGE ZENITH TELESCOPE

British Columbia, Canada (2003)

LARGE BINOCULAR TELESCOPE

Mt Graham, Arizona (2005)

GRAN TELESCOPIO CANARIAS

La Palma, Canary Islands,

Spain (2007)

.

KEPLER

Earth-trailing solar orbit (2009)



VERY LARGE TELESCOPE Cerro Paranal, Chile (1998-2000)

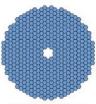








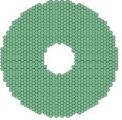
LARGE SYNOPTIC SURVEY TELESCOPE El Penón, Chile (planned 2020)



THIRTY METER TELESCOPE Mauna Kea, Hawaii (planned 2022)



GIANT MAGELLAN TELESCOPE Las Campanas Observatory, Chile (planned 2022/2025)



EUROPEAN EXTREMELY LARGE TELESCOPE Cerro Armazones, Chile (planned 2022)



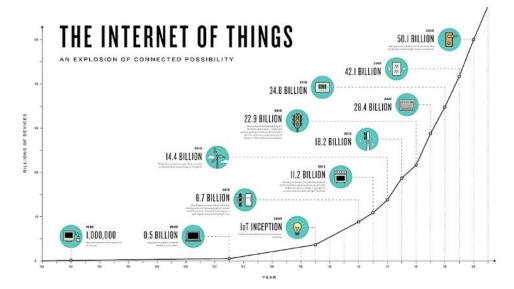
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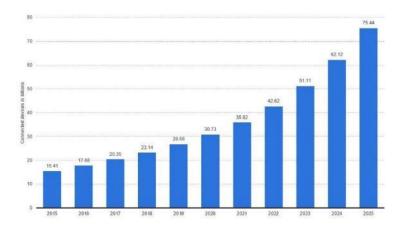
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Internet-of-Things



Internet of Things - number of connected devices worldwide 2015-2025

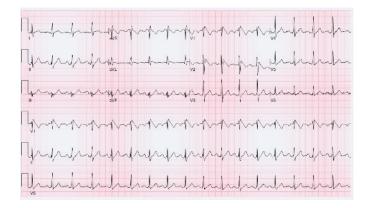
Internet of Things (IoT) connected devices installed base worldwide from 2015 to 2025 (in billions)

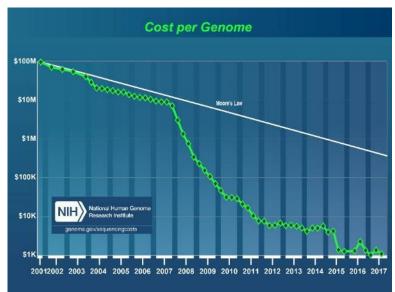


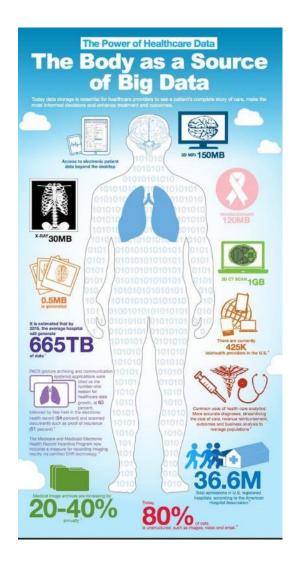




Biomedical signals







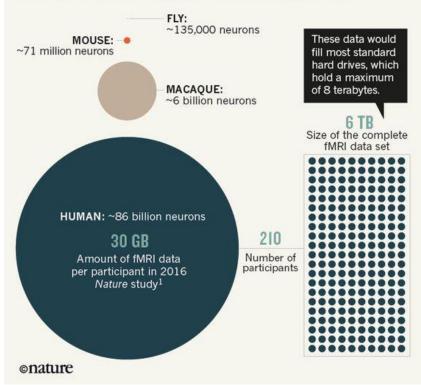




Neuroscience

BIG DATA BY THE NUMBERS

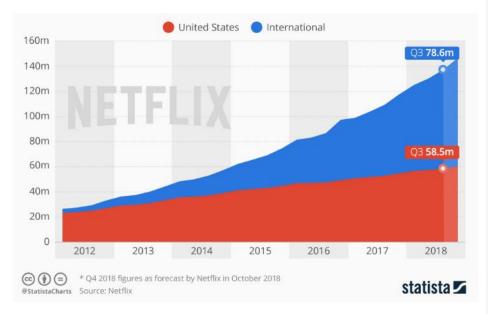
Mapping the brain presents an enticing challenge — and a computationally daunting one. Here's how many data one study last year generated.



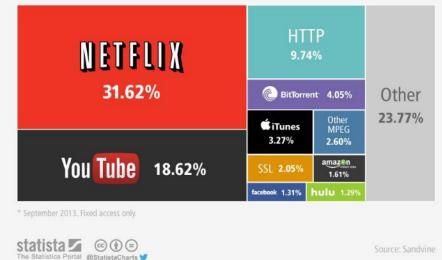




Imaging technologies



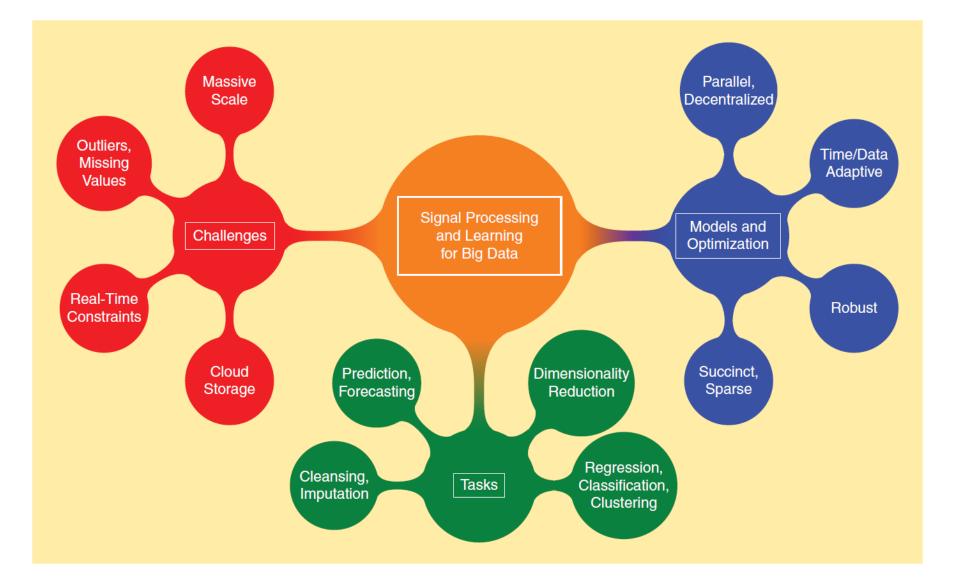
Netflix and YouTube Are America's Biggest Traffic Hogs Share of peak period downstream traffic in North America, by application*





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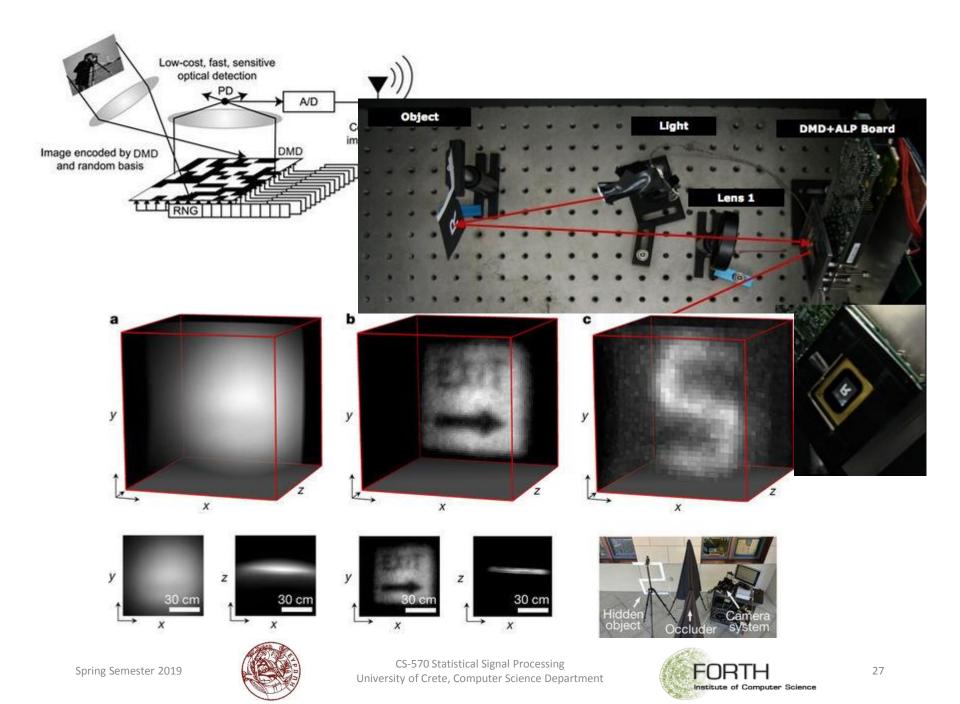


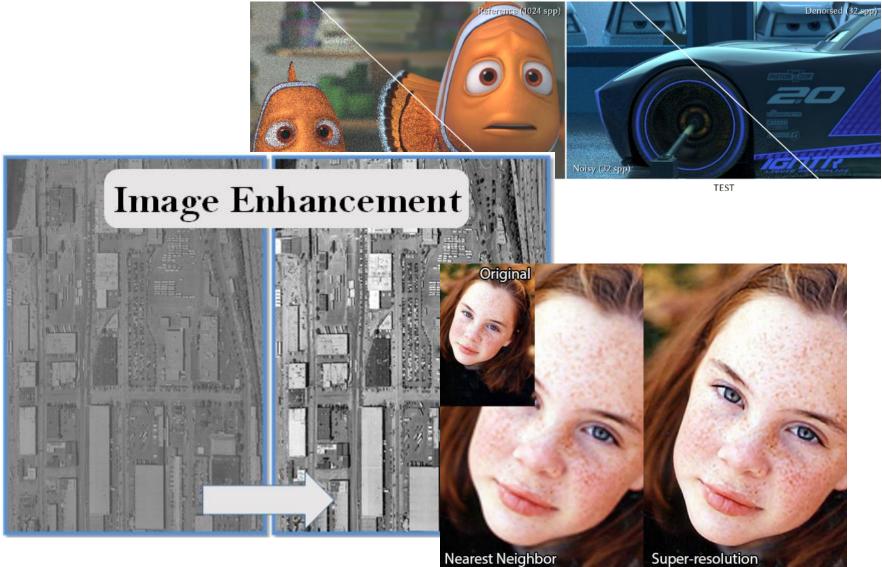
Fundamental Signal Processing

- Signal Sensing: Compressed Sensing
- Inverse problems: Signal Denoising, Enhancement
- Filtering
- Time-series modeling and prediction
- Modeling: dimensionality reduction





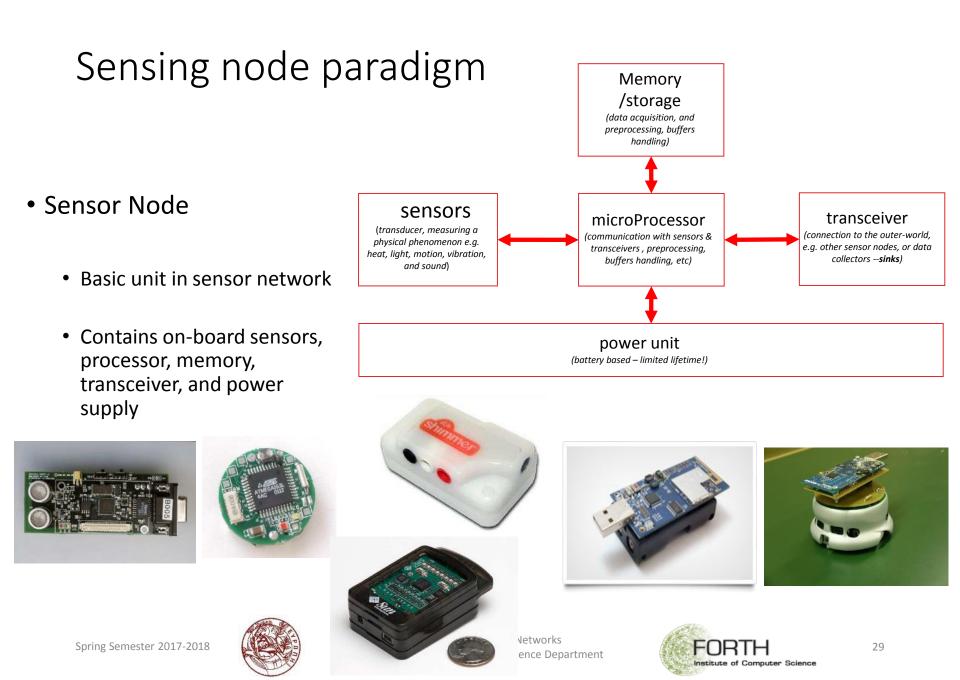




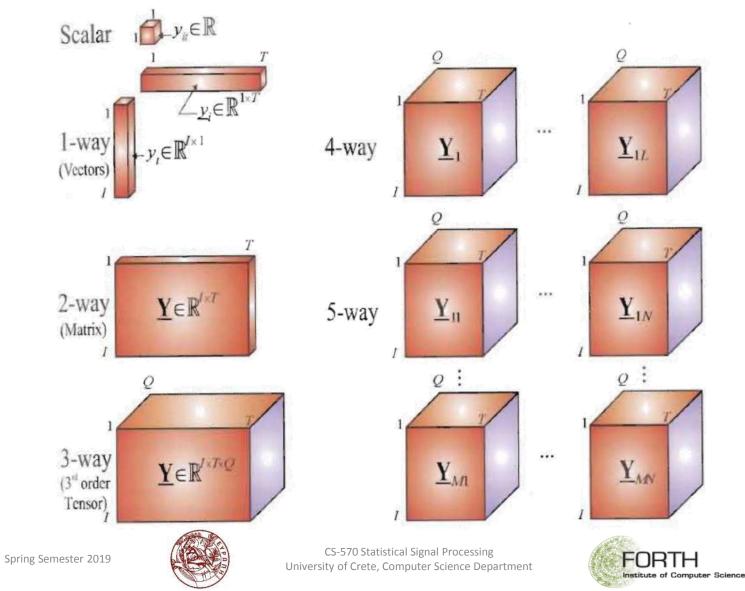
Super-resolution







Review of basic concepts



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Vectors • A column vector $\mathbf{v} \in \mathbb{R}^{n \times 1}$ where $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

• A row vector $\mathbf{v}^T \in \mathbb{R}^{1 imes n}$ where $\mathbf{v}^T = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix}$

T denotes the transpose operation

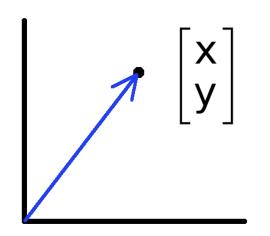


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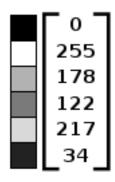


Vectors

- Vectors can represent an offset in 2D or 3D space
- Points are just vectors from the origin



- Data (pixels, gradients at an image keypoint, etc) can also be treated as a vector
- Such vectors don't have a geometric interpretation, but calculations like "distance" can still have value







Matrix

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• A matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ is an array of numbers with size $m \downarrow$ by $n \rightarrow$, i.e. m rows and n columns.

$\mathbf{A} =$	$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$	$a_{12} \\ a_{22}$	$a_{13} \\ a_{23}$		$\begin{bmatrix} a_{1n} \\ a_{2n} \end{bmatrix}$
			20		•
	a_{m1}	a_{m2}	a_{m3}	•••	a_{mn}

• If m=n , we say that ${f A}$ is square.

	г				_
	193	180	210	112	125
	189	8	177	97	114
	100	71	81	195	165
	167	12	242	203	181
	44	25	9	48	192
	1				



Basic Matrix Operations

- 7

Addition

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+1 & b+2 \\ c+3 & d+4 \end{bmatrix}$$

• Can only add a matrix with matching dimensions, or a scalar.

• Scaling

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + 7 = \begin{bmatrix} a+7 & b+7 \\ c+7 & d+7 \end{bmatrix}$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times 3 = \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix}$$

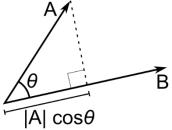




- Inner product (dot product) of vectors
 - Multiply corresponding entries of two vectors and add up the result
 - $x \cdot y$ is also $|x||y|\cos(the angle between x and y)$

$$\mathbf{x}^T \mathbf{y} = \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \sum_{i=1}^n x_i y_i$$
 (scalar)

• If B is a unit vector, then $A \cdot B$ gives the length of A which lies in the direction of B







Matrix Multiplication

$$C = AB$$
. $C_{i,j} = \sum_{k} A_{i,k} B_{k,j}$.

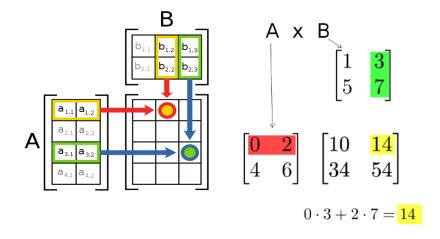
Properties

$$A(B+C) = AB + AC.$$

$$A(BC) = (AB)C.$$

$$(AB)^{\top} = B^{\top}A^{\top}$$

$$x^{\top}y = (x^{\top}y)^{\top} = y^{\top}x$$



- Powers
 - We can refer to the matrix product AA as A², and AAA as A³, etc.
 - Only square matrices can be multiplied that way





• Transpose
$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & 4 \\ 1 & 3 & 5 \end{bmatrix} (ABC)^T = C^T B^T A^T$$

• Determinant

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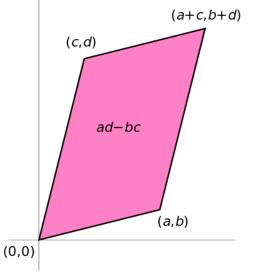
- $\det(\mathbf{A})$ returns a scalar
- Represents area of the parallelogram described by the vectors in the rows of the matrix

• For
$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \det(\mathbf{A}) = ad - bc$$

• Properties:
$$det(AB) = det(BA)$$

$$det(\mathbf{A}^{-1}) = \frac{1}{det(\mathbf{A})}$$
$$det(\mathbf{A}^{T}) = det(\mathbf{A})$$
$$det(\mathbf{A}) = 0 \Leftrightarrow \mathbf{A} \text{ is sing}$$

 $det(\mathbf{A}) = 0 \Leftrightarrow \mathbf{A}$ is singular







• Trace $tr(\mathbf{A}) = sum of diagonal elements$

$$\mathbf{tr}(\begin{bmatrix} 1 & 3\\ 5 & 7 \end{bmatrix}) = 1 + 7 = 8$$

Invariant to a lot of transformations

• Properties:
$$tr(AB) = tr(BA)$$

 $tr(A + B) = tr(A) + tr(B)$

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Special matrices

• Identity matrix **I**
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}$$

• Symmetric matrix
$$\mathbf{A}^T = \mathbf{A} \begin{bmatrix} 1 & 2 & 5 \\ 2 & 1 & 7 \\ 5 & 7 & 1 \end{bmatrix}$$

• Skew-symmetric matrix $\mathbf{A}^T = -\mathbf{A} \begin{bmatrix} 1 & -2 & -5 \\ 2 & 1 & -7 \\ 5 & 7 & 1 \end{bmatrix}$





Matrix Inverse

• Given a matrix **A**, its inverse **A**⁻¹ is a matrix such that **AA**⁻¹ = **A**⁻¹**A** = **I**

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

- Inverse does not always exist. If A⁻¹ exists, A is *invertible* or *non-singular*. Otherwise, it's *singular*.
- For matrices that are invertible

$$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$$
$$(\mathbf{A}\mathbf{B})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$
$$\mathbf{A}^{-T} \triangleq (\mathbf{A}^{T})^{-1} = (\mathbf{A}^{-1})^{T}$$



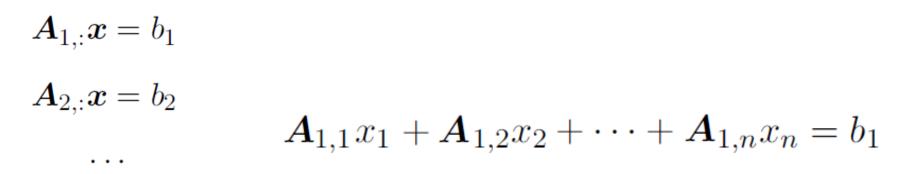


System of linear equations

$A \in \mathbb{R}^{m imes n}$ is a known matrix

 $oldsymbol{A} oldsymbol{x} = oldsymbol{b}$ $oldsymbol{b} \in \mathbb{R}^m$ is a known vector,

 $x \in \mathbb{R}^n$ is a vector of unknown variables



 $A_{m,:}x = b_m$





Solution of system

- Inverse of a matrix $A^{-1}A = I_n$
- Solution of systems of linear equation $oldsymbol{I}_n oldsymbol{x} = oldsymbol{A}^{-1}oldsymbol{b}$
- Provided A⁻¹ exists $x = A^{-1}b$.
- If both x and y are solutions then $z = \alpha x + (1 \alpha)y$ is also a solution for any real α



Ax = b

 $A^{-1}Ax = A^{-1}b$

Linear combinations

- Linear combination of some set of vectors $\{v(1), \ldots, v(n)\}$ is given by multiplying each vector v(i) by a corresponding scalar coefficient and adding the results: $\sum_{i} c_i v^{(i)}$
- The **span** of a set of vectors is the set of all points obtainable by linear combination of the original vectors.



Norms

$$||\boldsymbol{x}||_p = \left(\sum_i |x_i|^p\right)^{\frac{1}{p}} \quad p \in \mathbb{R}, p \ge 1$$

a norm is any function f that satisfies

- $f(\boldsymbol{x}) = 0 \Rightarrow \boldsymbol{x} = \boldsymbol{0}$
- $f(\boldsymbol{x} + \boldsymbol{y}) \leq f(\boldsymbol{x}) + f(\boldsymbol{y})$ (the triangle inequality)
- $\forall \alpha \in \mathbb{R}, f(\alpha \boldsymbol{x}) = |\alpha| f(\boldsymbol{x})$



Norms

• L₂ norm, also known as Euclidean norm

$$\|x\|_2 = \sqrt{\sum_i x_i^2}$$

- L₂₁ norm $||\boldsymbol{x}||_1 = \sum_i |x_i|$ Infinite norm (or max norm) $||\boldsymbol{x}||_{\infty} = \max_i |x_i|$
- Frobenius norm (Matrix norm)

$$||A||_F = \sqrt{\sum_{i,j} A_{i,j}^2},$$





Linear independence

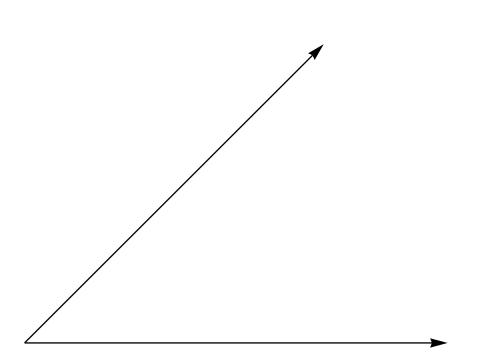
- Suppose we have a set of vectors $v_1, ..., v_n$
- If we can express v_1 as a linear combination of the other vectors $v_2...v_n$, then v_1 is linearly *dependent* on the other vectors.
 - The direction v₁ can be expressed as a combination of the directions v₂...v_n. (E.g. v₁ = .7 v₂ -.7 v₄)
- If no vector is linearly dependent on the rest of the set, the set is linearly *independent*.
 - Common case: a set of vectors $v_1, ..., v_n$ is always linearly independent if each vector is perpendicular to every other vector (and non-zero)



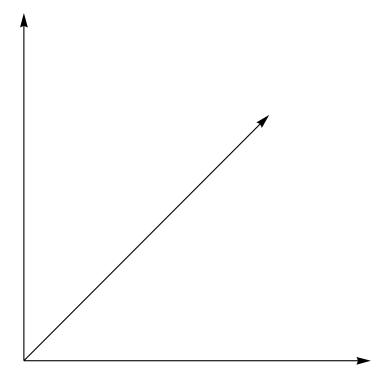


Linear independence

Linearly independent set



Not linearly independent





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